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Geomagnetism package notes

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Contents

1. Introduction

The package geomagnetim intends to serve pedagogical purpose rather to replace well established FORTRAN or C progam develloped by academic institutions. In contrast with these programs, which favor compactness and time execution, the geomagnetic package ,tentatively, focus on lisibility.
You can download geomagnetis using :

pip install geomagnetism

2. Geomagnetism calculation

As the terrestrial magnetic field obeys both $\nabla \mathbf{B} = 0$ and $\nabla \times \mathbf{B} = 0$, it can be shown that the magnetic field can be expressed as the gradient of a scalar potential V which satisfies the Laplace equation:

$$\Delta \mathrm{V} = 0 \tag{0.1}$$

For a spherical geometry the geomagnetic potential is given by the spherical harmonic expension (SH) (Stacey and Davis , Campbell 2007):

$$V(r,\theta,\phi,t) = a\sum_{n=1}^{N}\left(\frac{a}{r}\right)^{n+1}\sum_{m=0}^{n}\left[g_n^m(t)\cos(m\phi) + h_n^m(t)\sin(m\phi)\right]P_n^m\left(\cos(\theta)\right) \tag{0.2}$$

with:

$$P_n^m(x) = \begin{cases} \dfrac{1}{2^n n!}\dfrac{d^n}{dx^n}\left(x^2-1\right)^n & : m=0 \\ \sqrt{\dfrac{2(n-m)!}{(n+m)!}}\left(1-x^2\right)^m \dfrac{1}{2^n n!}\dfrac{d^{n+m}}{dx^n}\left(x^2-1\right)^n & : m>0 \end{cases} \tag{0.3}$$

g_n^m and h_n^m are the Gauss's coefficients. Note that the sum over n begins with the the value $n=1$ as the index $n=0$ would correspond to a monopole. The dipole, quadrupole, octupole,... contribution correspond to $n=1,2,3,...$ These coefficients varies with time and are tabulated (https://www.ngdc.noaa.gov/IAGA/vmod/igrf.html). The coefficient a is the mean radius of the earth (6371.2 km); r ,the radial distance from the center of the Earth ; θ, the geocentric colatitude ; ϕ, the east longitude measured from the Greewich. We note that the Condon-Shortley phase correction $(-1)^m$ is omitted in the definition of the associated Legendre polynomial and the polynomes are normalized using Schmidt quasi-normalization (Winch, Ivers et al. 2005). The relation $\mathbf{B}=-\nabla V$ leads to:

$$\boxed{\begin{cases} X_c = \mathbf{B}_x = -B_\theta = \dfrac{1}{r}\dfrac{\partial V}{\partial \theta} = \displaystyle\sum_{n=1}^{N}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}\left[g_n^m\cos(m\phi)+h_n^m\sin(m\phi)\right]\frac{dP_{(s),n}^m(\cos\theta)}{d\theta} \\ Y_c = \mathbf{B}_y = B_\phi = \dfrac{-1}{r\sin\theta}\dfrac{\partial V}{\partial \phi} = \displaystyle\sum_{n=1}^{N}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}m\left[g_n^m\sin(m\phi)-h_n^m\cos(m\phi)\right]\frac{P_{(s),n}^m(\cos\theta)}{\sin\theta} \\ Z_c = \mathbf{B}_z = -B_r = \dfrac{\partial V}{\partial r} = \displaystyle\sum_{n=1}^{N}(n+1)\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}\left[g_n^m\cos(m\phi)+h_n^m\sin(m\phi)\right]P_{(s),n}^m(\cos\theta) \end{cases}} \tag{0.4}$$

Where $\mathbf{B}_x$, $\mathbf{B}_y$, $\mathbf{B}_z$ are the field components respectively in the northward, eastward and downward directions. Theses components are expressed in the geocentric referential as recall by the index c

3. Spherical harmonics normalisation

Let define:

$$\begin{aligned} C_n^m(\theta,\phi) &\equiv P_{(s),n}^m\left(\cos\theta\right)\cos m\theta \quad : m=0,1,2\cdots n \\ S_n^m(\theta,\phi) &\equiv P_{(s),n}^m\left(\cos\theta\right)\sin m\theta \quad : m=1,2\cdots n \end{aligned} \tag{0.5}$$

the Schmidt quasi-normalization conditions reads (Winch, Ivers et al. 2005):

$$\begin{aligned} &\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} C_n^m(\theta,\phi)C_N^M(\theta,\phi)\sin\theta d\theta d\phi = \frac{1}{2n+1}\delta_n^N\delta_m^M \\ &\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} S_n^m(\theta,\phi)S_N^M(\theta,\phi)\sin\theta d\theta d\phi = \frac{1}{2n+1}\delta_n^N\delta_m^M \\ &\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} C_n^m(\theta,\phi)S_N^M(\theta,\phi)\sin\theta d\theta d\phi = 0 \quad :\forall n,N,m,M \end{aligned} \tag{0.6}$$

The associated Legendre polynomial value can be generated by the scipy function M,Mp=lpmn(M,N,x) where:

$$\underline{\underline{\mathbf{P}}} = \begin{bmatrix} P_0^0 & P_1^0 & \cdots & P_N^0 \\ 0 & P_1^1 & \cdots & P_N^1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & P_N^M \end{bmatrix} \tag{0.7}$$

with :

$$P_n^m(x) = \frac{(-1)^m}{2^n n!}\sqrt{\left(1-x^2\right)^m}\,\frac{d^{n+m}}{dx^n}\left(x^2-1\right)^n \tag{0.8}$$

To obtain the normalize Legendre polynomials $P_{(s),n}^m$ we use:

$$\underline{\underline{\mathbf{P}}}_{(s)} = \underline{\underline{\mathbf{P}}} \odot \underline{\underline{\mathbf{N}}} \tag{0.9}$$

where $\odot$ is the component-wise multiplication and :

$$\boxed{N_{n,m} = \begin{cases} (-1)^m\sqrt{\dfrac{\left(2-\delta_m^0\right)(n-m)!}{(n+m)!}} & : n-|m| \geq 0 \\ 0 \quad : n-|m| < 0 \end{cases}} \tag{0.10}$$

Note that other authors in geophysics use different normalization factors. Stacey (Stacey and Davis) use :

$$N_n'^m = \begin{cases} (-1)^m\sqrt{\left(2-\delta_m^0\right)(2m+1)\dfrac{(n-m)!}{(n+m)!}} & : |m| \leq n \\ 0 \quad : |m| > n \end{cases} \tag{0.11}$$

The function `Norm_Schimdt(m, n)` computes the normalisation matrix (0.10)

The function `Norm_ Stacey (m, n)` computes the normalisation matrix (0.11)

Exemples :

```
geo.Norm_Stacey(3,4)
>> array([[ 1.        ,  1.        ,  1.        ,  1.        ,  1.        ],
       [ 0.        , -1.73205081, -1.        , -0.70710678, -0.54772256],
       [ 0.        ,  0.        ,  0.64549722,  0.28867513,  0.16666667],
       [ 0.        ,  0.        ,  0.        , -0.13944334, -0.05270463]])


geo.Norm_ Schimdt (3,4)
>> array([[ 1.        ,  1.        ,  1.        ,  1.        ,  1.        ],
       [ 0.        , -1.        , -0.57735027, -0.40824829, -0.31622777],
       [ 0.        ,  0.        ,  0.28867513,  0.12909944,  0.0745356 ],
       [ 0.        ,  0.        ,  0.        , -0.05270463, -0.01992048]])
```

4. Geotetic to geocentric transformation

The computation of the geomagnetic field is done in a geocentric coordinate system. So if we provide The geotetic coordinates we have to convert them into geocentric ones. In the following we deduce the transformation relation used in the function `geodetic_to_geocentric`.

Ellipse equation

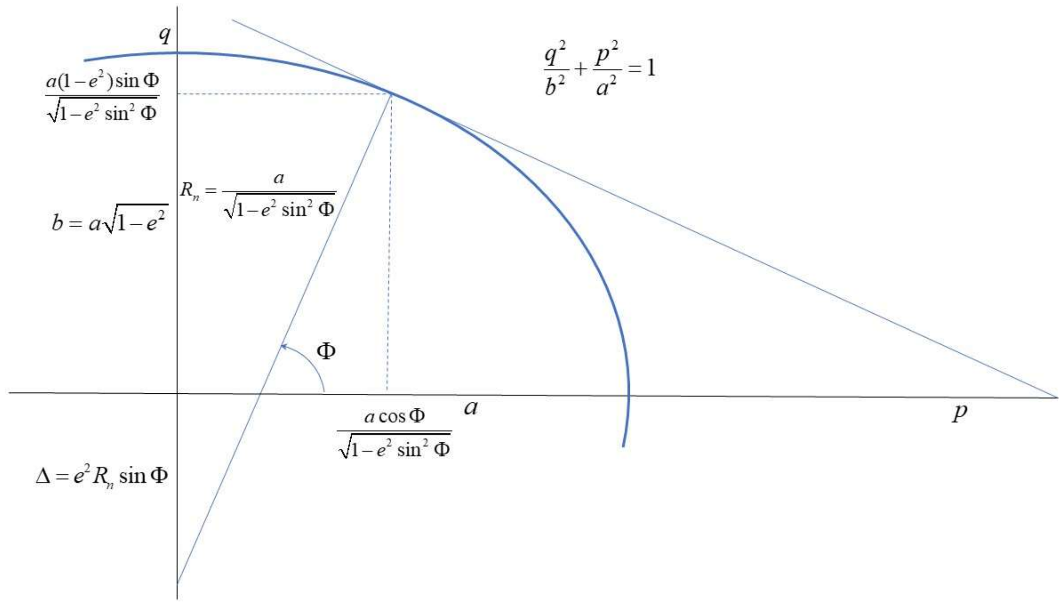


Figure 1 : Ellipse notation convention.

The equation of the ellipse reads :

$$\frac{q^2}{b^2}+\frac{p^2}{a^2}=1 \tag{0.12}$$

The geotetic latitude Φ can be be expressed through the derivative :

$$\cot\Phi=-\frac{dq}{dp}=\frac{b^2}{a^2}\frac{q}{p} \tag{0.13}$$

From (0.13) we can express q as :

$$q=p\frac{a^2}{b^2}\tan\Phi \tag{0.14}$$

Using (0.12) and (0.14) we obtain :

$$\begin{cases} p = \dfrac{a\cos\Phi}{\sqrt{1-e^2\sin^2\Phi}} \\ q = \dfrac{a(1-e^2)\sin\Phi}{\sqrt{1-e^2\sin^2\Phi}} \end{cases} \tag{0.15}$$

The prime vertical curvature radius R_n can be deduced from p as :

$$\begin{aligned} R_n &= \frac{p}{\cos\Phi} = \frac{a}{\sqrt{1-e^2\sin^2\Phi}} \\ R_n &= \frac{a^2}{\sqrt{a^2-(a^2-b^2)\sin^2\Phi}} \end{aligned} \tag{0.16}$$

Using the prime vertical curvature we can re-express p and q as :

$$\begin{cases} p = R_n\cos\Phi \\ q = (1-e^2)R_n\sin\Phi \end{cases} \tag{0.17}$$

Geotetic to geocentric transformation

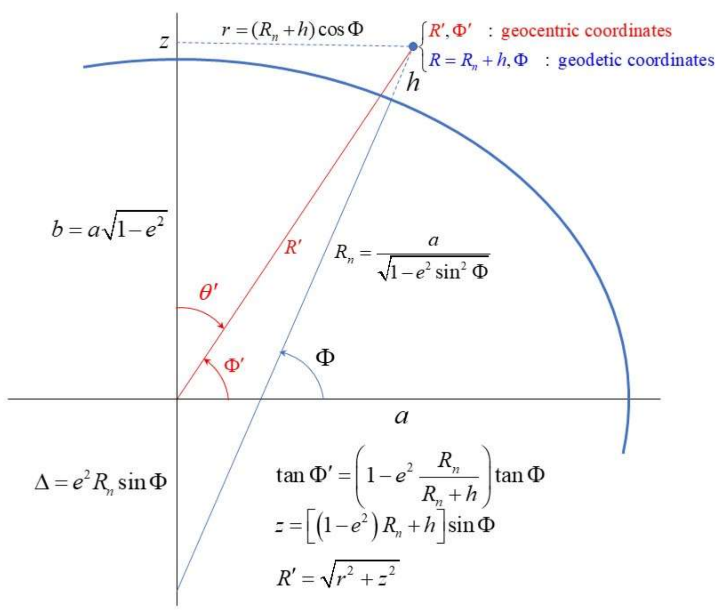


Figure 2 : Geotetic and geocentric notation convention.

Relation between the geotetic colatitude Φ and the geocentric colatitude Φ' :

Using the conventions of Figure 2 we have

$$\boxed{\frac{\tan\Phi'}{\tan\Phi}=\frac{z}{z+\Delta}=\frac{(1-e^2)R_n+h}{(1-e^2)R_n+h+e^2R_n}=1-e^2\frac{R_n}{R_n+h}} \tag{0.18}$$

The flattening is defined as follow:

$$f=\frac{a-b}{a} \tag{0.19}$$

The geodetic reference ellipsoids are specified by giving the reciprocal flattening f^{-1}

As usual the eccentricity reads:

$$\begin{aligned} e &= \sqrt{1-\frac{b^2}{a^2}}=\sqrt{f(2-f)} \\ 1-e^2 &= \frac{b^2}{a^2} \end{aligned} \tag{0.20}$$

We can express z and r as :

$$\begin{cases} z=(R_n+h)\sin\Phi-\Delta=\left[(1-e^2)R_n+h\right]\sin\Phi \\ r=(R_n+h)\cos\Phi \end{cases} \tag{0.21}$$

Using (0.21) the geocentric radius is equal to :

$$R'=\sqrt{z^2+r^2} \tag{0.22}$$

Using (0.22) we can express the geocentric radius using the semi major and the semi minor axis. We obtain (Peddie 1982) :

$$\boxed{R'^2=\frac{h^2+2h\sqrt{a^2-(a^2-b^2)\sin^2\Phi}+\left[a^4-(a^4-b^4)\sin^2\Phi\right]}{a^2-(a^2-b^2)\sin^2\Phi}} \tag{0.23}$$

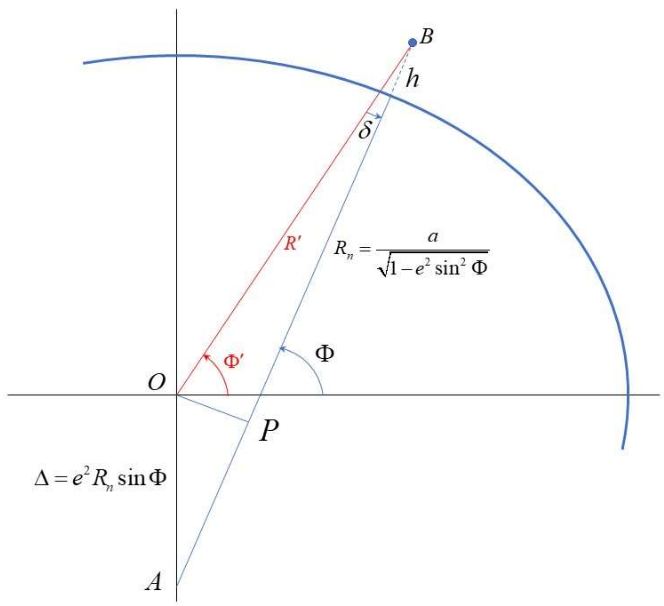


Figure 3 : Computation of $\cos\delta$ *and* $\sin\delta$ *.*

Using (0.21) and (0.22), the geocentric colatitude can be deduced from :

$$\cos\theta = \frac{z}{\sqrt{r^2+z^2}} = \frac{\sin\Phi}{\sqrt{\left[\frac{R_n+h}{(1-e^2)R_n+h}\right]^2\cos^2\Phi+\sin^2\Phi}} \tag{0.24}$$

The relation (0.24) can be expressed using the semi major and minor axis. Using (0.16), (0.20) we obtain (Peddie, 1982) :

$$\cos\theta = \frac{\sin\Phi}{\sqrt{c\cos^2\Phi+\sin^2\Phi}}; \quad c = \left[\frac{a^2+h\sqrt{a^2-(a^2-b^2)\sin^2\Phi}}{b^2+h\sqrt{a^2-(a^2-b^2)\sin^2\Phi}}\right]^2 \tag{0.25}$$

Using the triangle AOB of the Figure 3 we obtain the relation :

$$\Delta\sin\Phi + R'\cos\delta = R_n + h \tag{0.26}$$

After rearanging :

$$\cos\delta = \frac{1}{R'}\left[h+R_n(1-e^2)\right] \tag{0.27}$$

Using (0.16) we have :

$$\boxed{\cos\delta = \frac{1}{R'}\left[h+\frac{a^2}{R_n}\right]} \tag{0.28}$$

The length of OP is equal to

$$R' \sin \delta = \Delta \cos \Phi \tag{0.29}$$

So :

$$\sin \delta = \frac{R_n}{R'} e^2 \cos \Phi \sin \Phi \tag{0.30}$$

Computational aspect

The function `geodetic_to_geocentric(ellipsoid, co_latitude, height)` computes the geocentric and the geocentric colatitude using respectively (0.22) and (0.18). The angle

$$\delta = \theta' - \theta \tag{0.31}$$

between the geocentric and the geotetic colattitude is also computed (Figure 4).

The routine uses the tuple ellipsoid = $\left(a, f^{-1}\right)$. Depending on the selected convention ellipsoid can be set to

GRS80 = geo.geomagnetism.GRS80

WGS84 = geo.geomagnetism.WGS84

Exemple :

```
r_geocentric, co_latitude_geocentric, delta =
geo.geodetic_to_geocentric(geo.geomagnetism.WGS84 , 170, 100_000)
```

```
>> (6457402.34844737, 2.965925285681976, -0.0011344427083841424)
```

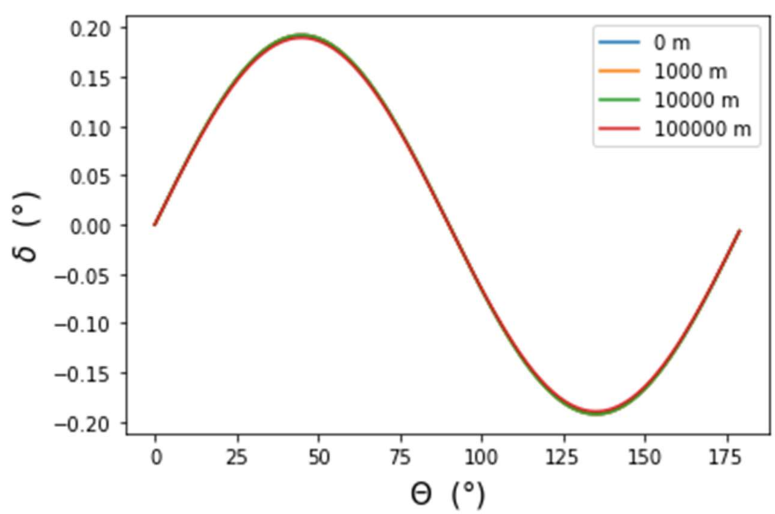


Figure 4 : Variation of δ versus the geotetic colatitude θ and the height h .

The function `geodetic_to_geocentric_IGRF13(ellipsoid, co_latitude, height)` is a translation of a FORTRAN (https://www.ngdc.noaa.gov/IAGA/vmod/igrf.html) where the authors use (0.23), (0.28), (0.30) to compute the geocentric radius the cosine and sine of the angle δ .

The function use the tuple ellipoid=(a,b). Depending on the selected convention ellipsoid can be set to

GRS80_ = geo.geomagnetism.GRS80_
WGS84 _= geo.geomagnetism.WGS84 _

5. Base transformation

Passing from geocentric to geotetic referential the magnetic field undergoes the following transfomation :

$$\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}_{\text{geodetic}} = \begin{bmatrix} \cos\delta & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\delta & 0 & \cos\delta \end{bmatrix} \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}_{\text{geocentric}} \quad (0.32)$$

with $\delta = \theta' - \theta = \Phi - \Phi'$ (Figure 5). Using Peddie notation (Peddie, 1982) we have :

$$\boxed{\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{bmatrix} \cos\delta & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\delta & 0 & \cos\delta \end{bmatrix} \begin{pmatrix} X_c \\ Y_c \\ Z_c \end{pmatrix}} \quad (0.33)$$

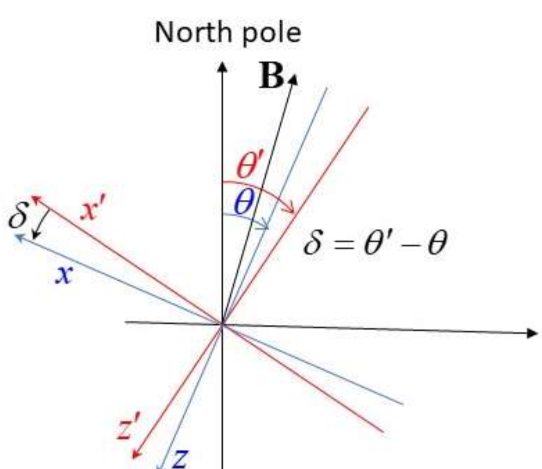


Figure 5 : Variation of δ versus the geotetic colatitude θ and the height h .

Using the notations of the Figure 6, the geomagnetic horizontal intensity H , total intensity F , declination D and inclination I can be obtained from :

$$\begin{cases} H = \sqrt{X^2 + Y^2} \\ F = \sqrt{H^2 + Y^2} \\ D = \operatorname{atan} 2(Y, X) \\ I = \operatorname{atan} 2(Z, H) \end{cases} \tag{0.34}$$

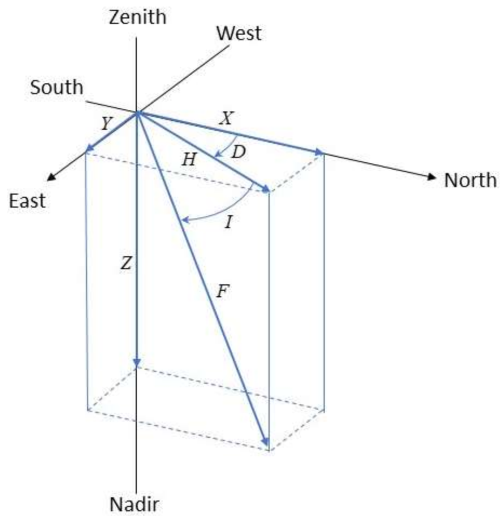


Figure 6 : Geomagnetic conventions and notations from (Chullia, 2020)

6. Geomagnetic field at North and South pole

North pole

For $\theta = 0$ we have to evaluate

$$\frac{dP^m_{(s),n}(\cos\theta)}{d\theta}, \frac{P^m_{(s),n}(\cos\theta)}{\sin\theta}, P^m_{(s),n}(\cos\theta) \tag{0.35}$$

The associate Legendre polynomial can be defined as:

$$P_n^m(x) = (1-x^2)^{\frac{m}{2}} \frac{d^m}{dx^m} P_n(x) \tag{0.36}$$

From (0.36) we deduce that $\frac{P_n^m(\cos\theta)}{\sin\theta}$ is not null if and only if $m = 1$. If that condition is fullfilled we have :

$$\frac{P_n^1(\cos\theta)}{\sin\theta} \xrightarrow[\theta \to 0]{} \frac{d}{dx} P_n(x)\Big|_{x=1} \tag{0.37}$$

From the differential equation :

$$(1-x^2)\frac{d^2}{dx^2}P_n(x)-2x\frac{d}{dx}P_n(x)+n(n+1)P_n(x)=0 \tag{0.38}$$

We deduce :

$$\frac{d}{dx}P_n(1)=\frac{n(n+1)}{2}, \qquad \frac{d}{dx}P_n(-1)=(-1)^{n-1}\frac{n(n+1)}{2} \tag{0.39}$$

Taking into account (0.39) and the Schmidt normalisation coefficient (0.10) we obtain :

$$\frac{P^m_{(s),n}(\cos\theta)}{\sin\theta}\xrightarrow[\theta\to 0]{}\delta_1^m\sqrt{\frac{n(n+1)}{2}} \tag{0.40}$$

From (0.36) we deduce :

$$P^m_{(s),n}(1)=\delta_m^0 \tag{0.41}$$

Concerning the derivative of the Legendre polynomials we have at the right boundary:

$$\left.\frac{dP_n^m(x)}{dx}\right|_{x=1}=\begin{cases}\frac{n(n+1)}{2} & \text{if } m=0\\ \infty & \text{if } m=1\\ -\frac{(n-1)n(n+1)(n+2)}{4} & \text{if } m=2\\ 0 & \text{if m=3,4,}\dots\end{cases} \tag{0.42}$$

Using (0.42) we obtain :

$$\left.\frac{dP_n^m(\cos\theta)}{d\theta}\right|_{\theta=0}=\delta_1^m\sqrt{\frac{n(n+1)}{2}} \tag{0.43}$$

Aggregating (0.41), (0.40), (0.43) we obtain :

$$\begin{cases}X_c(0)=\sum_{n=1}^{N}\left(\frac{a}{r}\right)^{n+2}\sqrt{\frac{n(n+1)}{2}}g_n^1\cos(\phi)+\sum_{n=1}^{N}\left(\frac{a}{r}\right)^{n+2}\sqrt{\frac{n(n+1)}{2}}h_n^1\sin(\phi)\\ Y_c(0)=\sum_{n=1}^{N}\left(\frac{a}{r}\right)^{n+2}\sqrt{\frac{n(n+1)}{2}}g_n^1\sin(\phi)-\sum_{n=1}^{N}\left(\frac{a}{r}\right)^{n+2}\sqrt{\frac{n(n+1)}{2}}h_n^1\cos(\phi)\\ Z_c(0)=\sum_{n=1}^{N}(n+1)\left(\frac{a}{r}\right)^{n+2}g_n^0\end{cases} \tag{0.44}$$

South pole

For or for $\theta = \pi$ we have :

$$P_{(s),n}^{m}(-1) = \delta_m^0 (-1)^n \quad (0.45)$$

$$\frac{P_{(s),n}^{m}(\cos\theta)}{\sin\theta} \xrightarrow[\theta\to\pi]{} \delta_1^m (-1)^n \sqrt{\frac{n(n+1)}{2}} \quad (0.46)$$

$$\left.\frac{dP_n^m(x)}{dx}\right|_{x=-1} = \begin{cases} (-1)^{n+1}\dfrac{n(n+1)}{2} & \text{if } m=0 \\ (-1)^n \infty & \text{if } m=1 \\ (-1)^n \dfrac{(n-1)n(n+1)(n+2)}{4} & \text{if } m=2 \\ 0 & \text{if m=3,4,...} \end{cases} \quad (0.47)$$

Aggregating (0.45), (0.46), (0.47) we have :

$$\begin{cases} X_c(0) = \sum_{n=1}^{N}\left(-\frac{a}{r}\right)^{n+2}\sqrt{\frac{n(n+1)}{2}}g_n^1\cos(\phi) + \sum_{n=1}^{N}\left(-\frac{a}{r}\right)^{n+2}\sqrt{\frac{n(n+1)}{2}}h_n^1\sin(\phi) \\ Y_c(0) = \sum_{n=1}^{N}\left(-\frac{a}{r}\right)^{n+2}\sqrt{\frac{n(n+1)}{2}}g_n^1\sin(\phi) - \sum_{n=1}^{N}\left(-\frac{a}{r}\right)^{n+2}\sqrt{\frac{n(n+1)}{2}}h_n^1\cos(\phi) \\ Z_c(0) = -\sum_{n=1}^{N}(n+1)\left(-\frac{a}{r}\right)^{n+2} g_n^0 \end{cases} \quad (0.48)$$

In geomagnetism we define the constant EPS = 1.0e-5° such that :

- If EPS < colatitude < 180- EPS we apply (0.4) `field_computation(r_a, M, Mp, phi, theta, dic_g, dic_h, N, mat_rot)`

- If colatitde < EPS we apply (0.44) `field_computation_pole(r_a, phi, theta, dic_g, dic_h, N, mat_rot, EPS)`

- If colatitude > 180 – EPS we apply **Erreur ! Source du renvoi introuvable.** `field_computation_pole(r_a, phi,`
- `theta, dic_g, dic_h, N, mat_rot, EPS)`

The Figure 7 shows the tiny discontinuities in the geomagnetic fields around EPS.

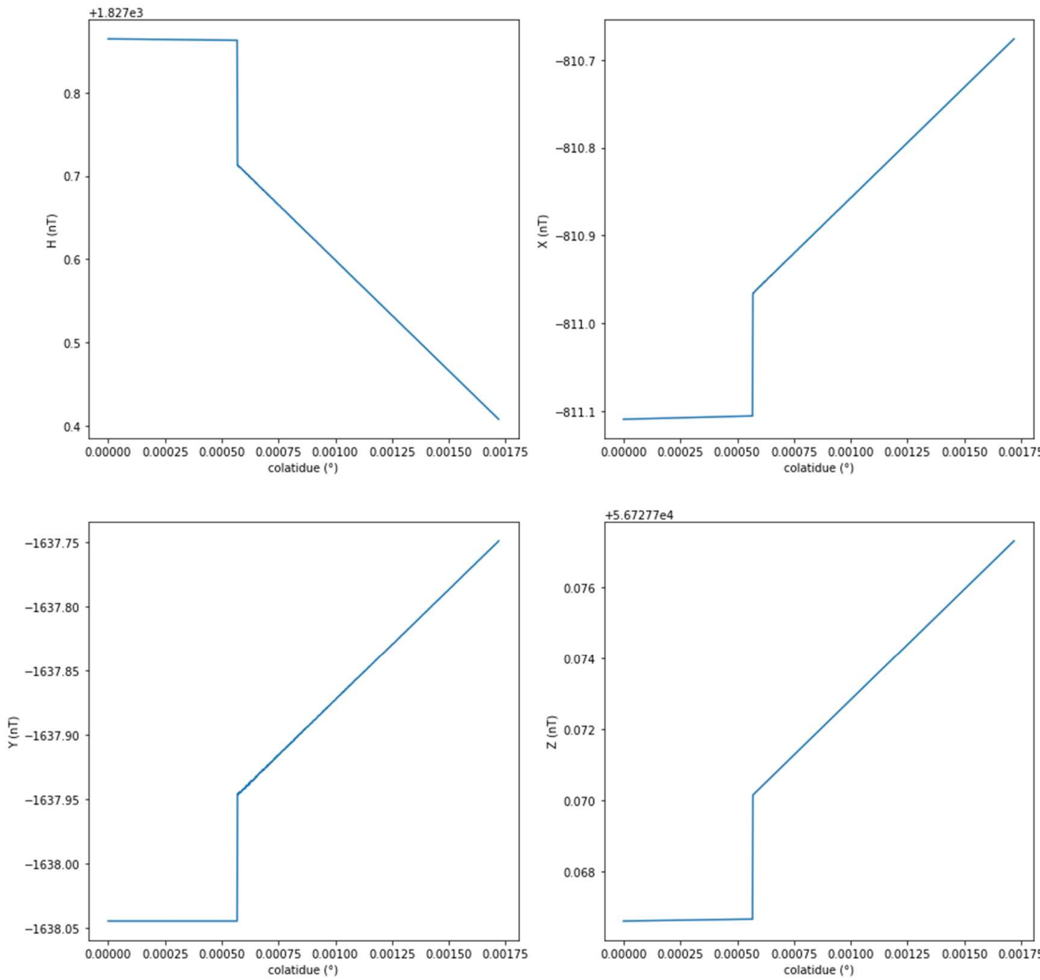


Figure 7 : Discontinuities in the field computation at colatitude = EPS.

7. Spherial harmonic coefficients

Gauss coefficients

The main ingredient for the computing of the geomagnetic field are the spherical harmonic coefficients g_m^n and h_n^m . These coefficients depend on the geocentic latitude on the longitude, and on the time. The package `geomagnetism` provide several functions to read tabulated cofficients

```
import geomagnetism as geo
file = "IGRF13.COF" # downloaded from
https://www.ngdc.noaa.gov/IAGA/vmod/coeffs/igrf13coeffs.txt
dic_dic_h, dic_dic_g, dic_dic_SV_h, dic_dic_SV_g, dic_N, Years=
geo.read_IGRF13_COF(file)
m=1
n=3
years="1965"
h = dic_dic_h[years][(m,n)]
print(f'Spherial harmonic coefficients h(m={m},n={n}) for year = {years} :{h}')
>>Spherial harmonic coefficients h(m=1,n=3) for year = 1965 :-404.0
```

Tableau 1 : Gauss coefficients from different sources.

File name	Deb	End	Source
WMM_2020.COF	2020	2020	https://www.ngdc.noaa.gov/geomag/WMM/soft.shtml
WMM_2015.COF	2015	2015	https://www.ngdc.noaa.gov/geomag/WMM/soft.shtml
IGRF13.COF	1900	2020	https://www.ngdc.noaa.gov/IAGA/vmod/coeffs/igrf13coeffs.txt
IGRF13coeffs.txt	1900	2020	https://www.ngdc.noaa.gov/IAGA/vmod/coeffs/igrf13coeffs.txt
FORTRAN_1900_1995.txt	1900	1995	https://www.ngdc.noaa.gov/IAGA/vmod/igrf13.f

IGRF13.COF and IGRF13coeffs.txt contain the same data with a different format.

Gauss coefficients secular variation

As the Gauss coefficients of the geomagnetic field vary with time their secular variation $\dot{g}_n^m$ and $\dot{h}_n^m$ expressed in nT/year are tabulated. We have :

$$\begin{cases} g_m^n(t) = g_m^n(t_0) + \dot{g}_m^n(t_0)(t-t_0) \\ h_m^n(t) = h_m^n(t_0) + \dot{h}_m^n(t_0)(t-t_0) \end{cases} \tag{0.49}$$

The secular vaiation are stored in the dictionaries `dic_dic_SV_h, dic_dic_SV_g`. Using these coefficients it is straightforward to compute the secular variation of the geomagnetic field componants. We have (Chullia, 2020):

$$\boxed{\begin{cases} \dot{X}_c = \dot{\mathbf{B}}_x = -\dot{B}_\theta = \dfrac{1}{r}\dfrac{\partial \dot{V}}{\partial \theta} = \displaystyle\sum_{n=1}^{N}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}\left[\dot{g}_n^m\cos(m\phi)+\dot{h}_n^m\sin(m\phi)\right]\frac{dP_{(s),n}^m(\cos\theta)}{d\theta} \\ \dot{Y}_c = \dot{\mathbf{B}}_y = \dot{B}_\phi = \dfrac{-1}{r\sin\theta}\dfrac{\partial \dot{V}}{\partial \phi} = \displaystyle\sum_{n=1}^{N}\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}m\left[\dot{g}_n^m\sin(m\phi)-\dot{h}_n^m\cos(m\phi)\right]\frac{P_{(s),n}^m(\cos\theta)}{\sin\theta} \\ \dot{Z}_c = \dot{\mathbf{B}}_z = -\dot{B}_r = \dfrac{\partial \dot{V}}{\partial r} = \displaystyle\sum_{n=1}^{N}(n+1)\left(\frac{a}{r}\right)^{n+2}\sum_{m=0}^{n}\left[\dot{g}_n^m\cos(m\phi)+\dot{h}_n^m\sin(m\phi)\right]P_{(s),n}^m(\cos\theta) \end{cases}}$$

(0.50)

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} = \begin{bmatrix} \cos\delta & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\delta & 0 & \cos\delta \end{bmatrix}\begin{pmatrix} \dot{X}_c \\ \dot{Y}_c \\ \dot{Z}_c \end{pmatrix} \tag{0.51}$$

Using (0.34) we obtain[1] :

[1] We use $\dfrac{\partial}{\partial x}\operatorname{atan}2(y,x) = -\dfrac{y}{x^2+y^2}$, $\dfrac{\partial}{\partial y}\operatorname{atan}2(y,x) = \dfrac{x}{x^2+y^2}$

a)

$$\begin{cases} \dot{H} = \dfrac{X\dot{X} + Y\dot{Y}}{H} \\ \dot{F} = \dfrac{X\dot{X} + Y\dot{Y} + Z\dot{Z}}{F} \\ \dot{I} = \dfrac{H\dot{Z} - Z\dot{H}}{F^2} \\ \dot{D} = \dfrac{X\dot{Y} - Y\dot{X}}{H^2} \end{cases} \tag{0.52}$$

8. Exemples, benchmark

Using the latitude, longitude, time and height above the ellipsoid given in Tableau 2, the shows a results comparison of the geomagnetic field element obtained by both the geomagnetism package and those extract from the Table 3b High-precision numerical example (Chullia, 2020).

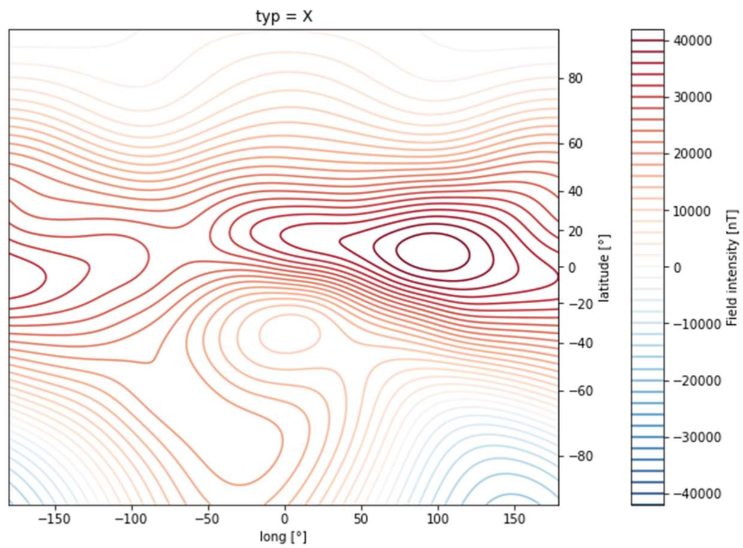
Tableau 2 : parameters values used for the benchmark

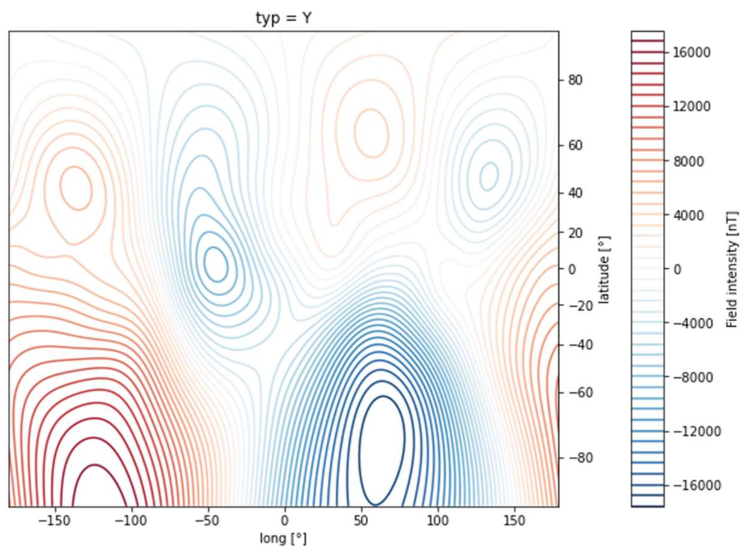
Time	2022.5000 0000	yr
Height-above-Ellipsoid	100.0000 0000	km
Latitude	-80.0000 0000	deg
Longitude	240.0000 0000	deg

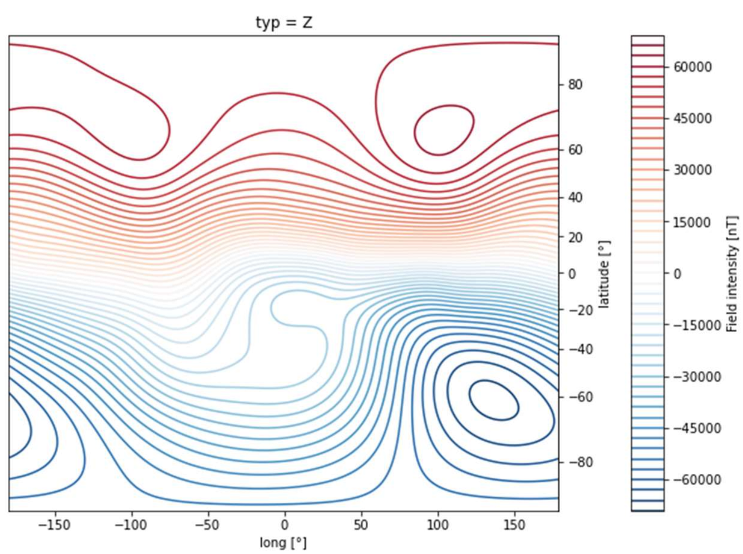
Tableau 3 : comparison of the geomagnetic field element obtained by both the geomagnetism package and those extract from the Table 3b High-precision numerical example (Chullia, 2020)

	geomagnetism	WMM test value for 2020	relative error	unit
D	68.5538905649841581	68.5538905643625469	0.0000000000090675	°
Dd	-0.0921756586115966	-0.0921756567227481	0.0000000204918372	°/yr
F	52235.3588449608359952	52235.3588449607996154	0.0000000000000007	nT
Fd	-78.0481471752882783	-78.0481471753000022	-0.0000000000001502	nT/yr
H	15904.1391483372881339	15904.1391483372990479	-0.0000000000000007	nT
Hd	11.5518244234884691	11.5518244234999994	-0.0000000000009981	nT/yr
I	-72.2736738948613606	-72.2736738961215934	-0.0000000000174370	°
Id	0.0406672571772073	0.0406672551433468	0.0000000500122385	°/yr
X	5814.9658886214674567	5814.9658886215001985	-0.0000000000000056	nT
Xd	28.0381961826633521	28.0381961826999984	-0.0000000000013070	nT/yr
Y	14802.9663839327658934	14802.9663839328004542	-0.0000000000000023	nT
Yd	1.3970624624335226	1.3970624624000001	0.0000000000239950	nT/yr
Z	-49755.3119939183306997	-49755.3119939183015958	0.0000000000000006	nT
Zd	85.6309533031287344	85.6309533031000001	0.0000000000003356	nT/yr

Using the `function grid_geomagnetic(colatitudes, longitudes, height=0, Date={"mode":"dec","year":2020.0})` we build two xarrays containing the cartography of the geomagnetic field components. The Figure 8 shows the isolines of geomagnetic field components. We have used Miller projection. These results are in good agreement with those of Chullia (Chullia, 2020).


typ = X
latitude [°]
long [°]
Field intensity [nT]
40000
30000
20000
10000
0
−10000
−20000
−30000
−40000
80
60
40
20
0
−20
−40
−60
−80
−150
−100
−50
0
50
100
150


typ = Y
latitude [°]
long [°]
Field intensity [nT]
16000
12000
8000
4000
0
−4000
−8000
−12000
−16000
80
60
40
20
0
−20
−40
−60
−80
−150
−100
−50
0
50
100
150


typ = Z
latitude [°]
long [°]
Field intensity [nT]
60000
45000
30000
15000
0
−15000
−30000
−45000
−60000
80
60
40
20
0
−20
−40
−60
−80
−150
−100
−50
0
50
100
150

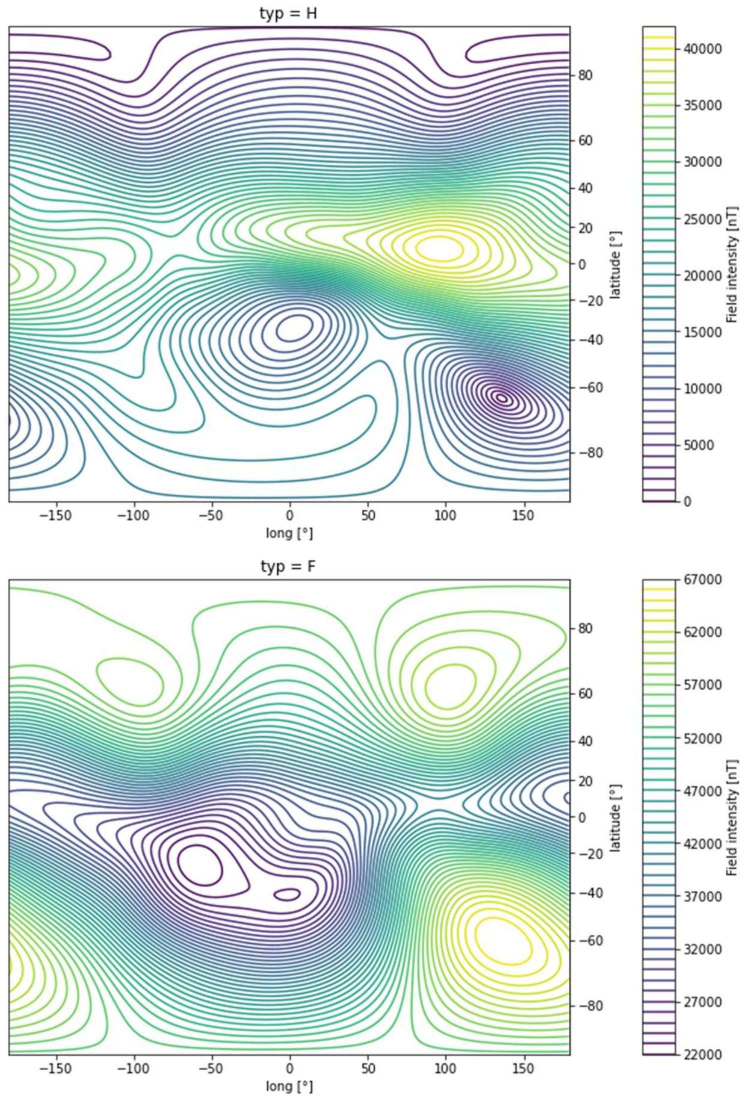


Figure 8 : Isovalues of the magnitude of the geomagnetic field componants X, Y, Z and intensities H, F. We use the convention of Figure 6, and the Miller projection.

The Figure 9 shows the isovalues of the declination and inclination of the geomagnetic field.

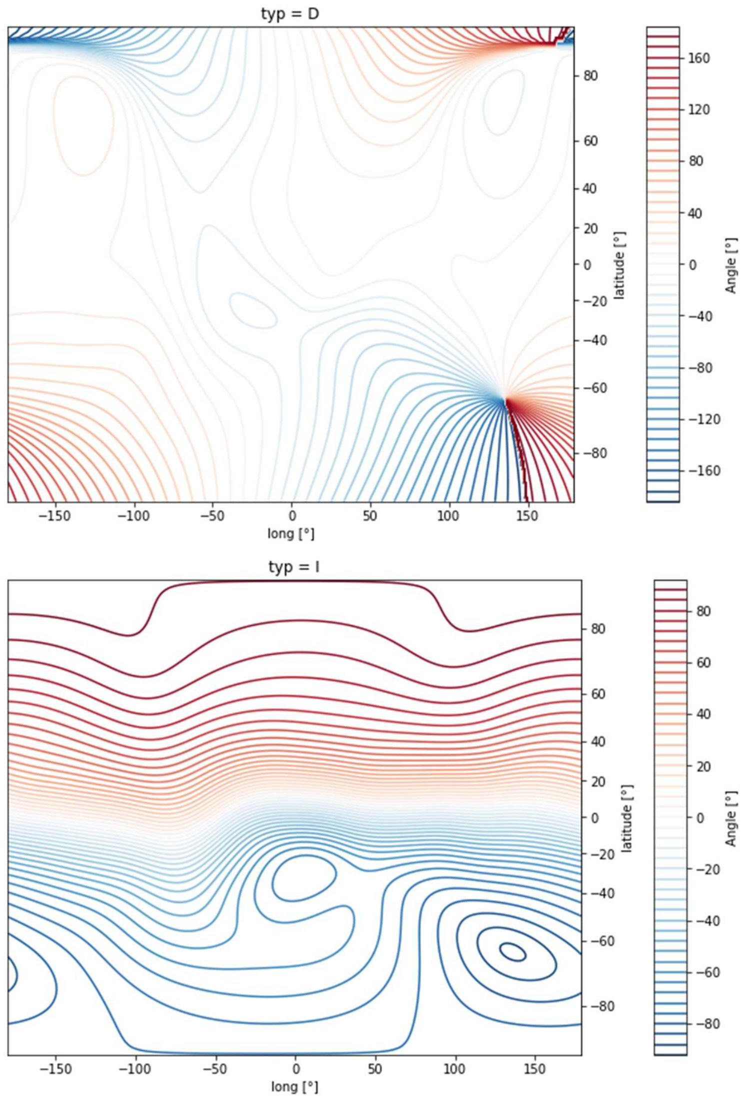
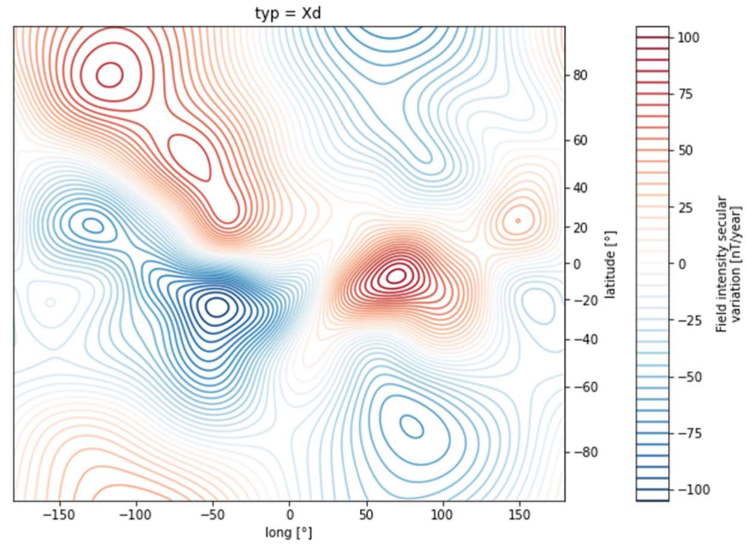
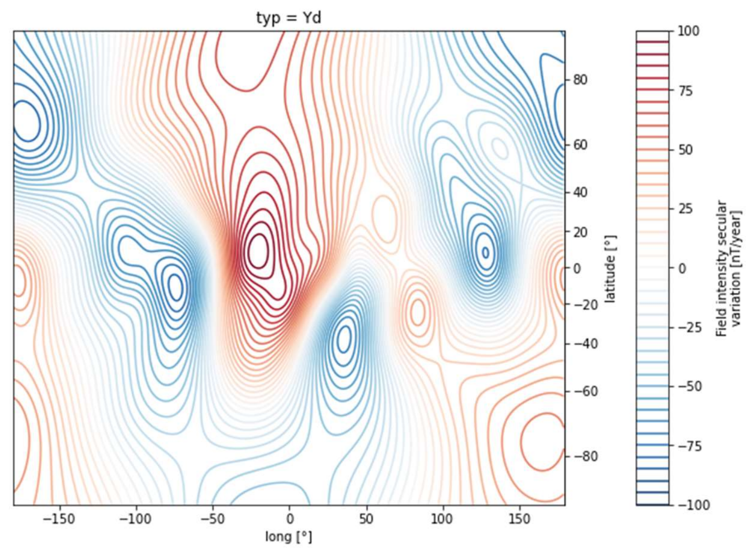
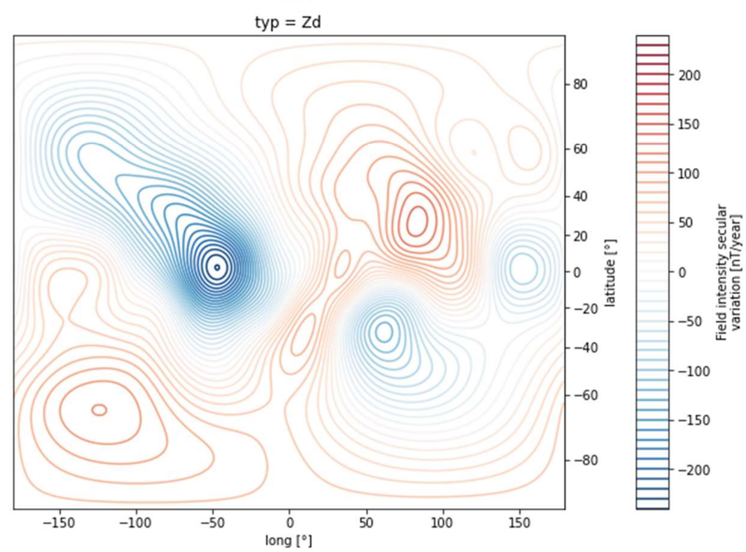


Figure 9 : Isovalues of the magnitude of the geomagnetic declination, D, and inclination, I. We use the convention of Figure 6, and the Miller projection.


typ = Xd
long [°]
latitude [°]
Field intensity secular variation [nT/year]
-150 -100 -50 0 50 100 150
80 60 40 20 0 -20 -40 -60 -80
100 75 50 25 0 -25 -50 -75 -100


typ = Yd
long [°]
latitude [°]
Field intensity secular variation [nT/year]
-150 -100 -50 0 50 100 150
80 60 40 20 0 -20 -40 -60 -80
100 75 50 25 0 -25 -50 -75 -100


typ = Zd
long [°]
latitude [°]
Field intensity secular variation [nT/year]
-150 -100 -50 0 50 100 150
80 60 40 20 0 -20 -40 -60 -80
200 150 100 50 0 -50 -100 -150 -200

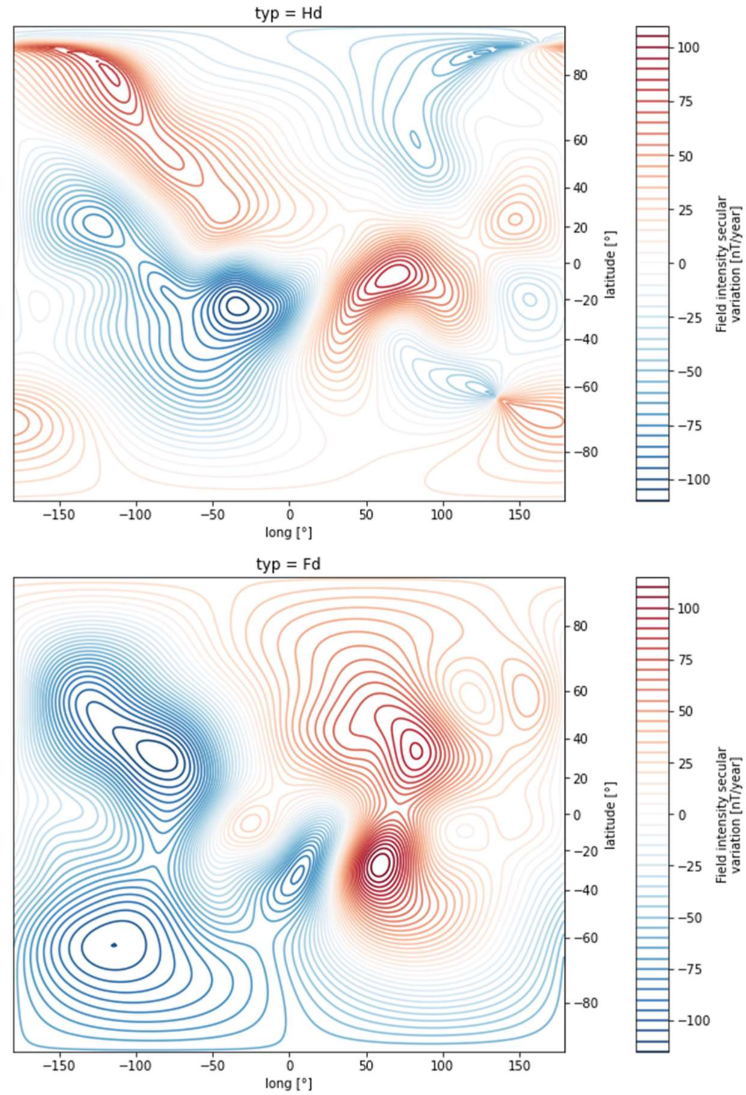


Figure 10 : Isovalues of the magnitude of the secular variation of the geomagnetic field componants X, Y, Z and intensities H, F. We use the convention of Figure 6, and the Miller projection.

The Figure 9 shows the isovalues of the declination and inclination of the geomagnetic field.

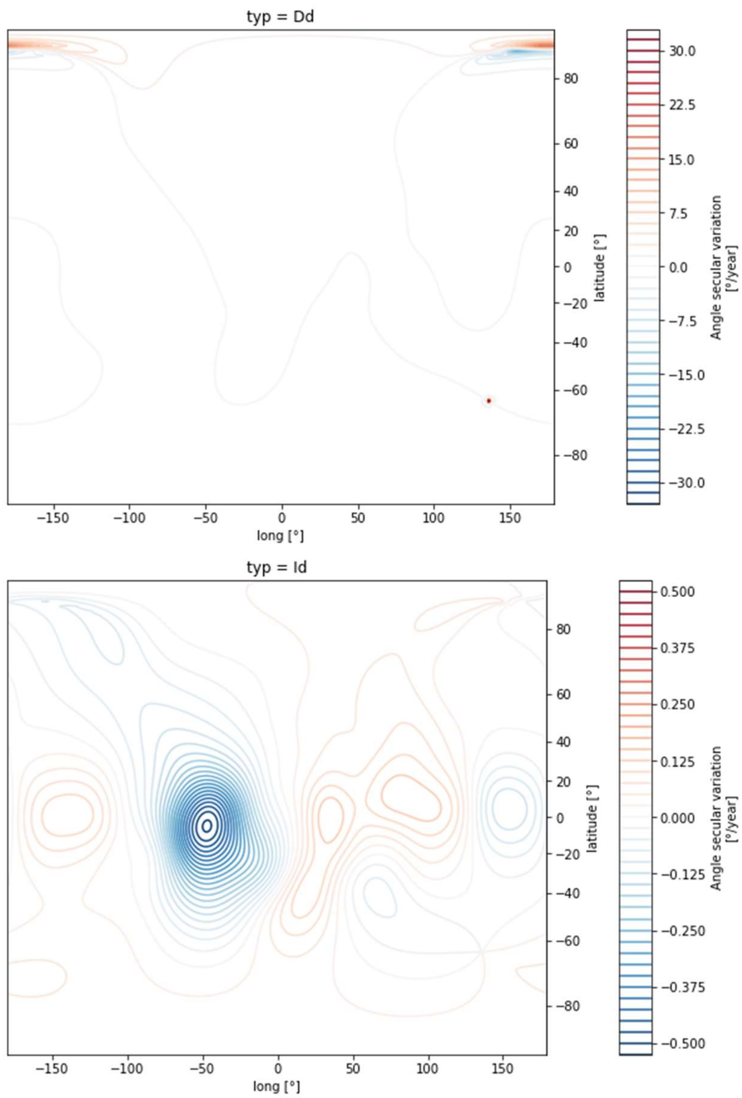


Figure 11 : Isovalues of the magnitude of thesecular variation of the geomagnetic declination, D, and inclination, I. We use the convention of Figure 6, and the Miller projection.

9. Other programs

- In line calculators

https://www.ngdc.noaa.gov/geomag/calculators/magcalc.shtml

http://wdc.kugi.kyoto-u.ac.jp/igrf/gggm/index.html

permits the computation of the magnetic declination.

- a FORTRAN code available can be downloaded from :

https://www.ngdc.noaa.gov/IAGA/vmod/igrf.html .

Qote: "This code is is a synthesis routine for the 13th generation IGRF as agreed in December 2019 by IAGA Working Group V-MOD. It is valid 1900.0 to 2025.0 inclusive. Values for dates from 1945.0 to 2015.0 inclusive are definitive, otherwise they are non-definitive. Reference radius remains as 6371.2 km - it is NOT the mean radius (= 6371.0 km) but 6371.2 km is what is used in determining the coefficients.'

- a C code available along with the Geomag 7.0 software (Windows version) :

https://www.ngdc.noaa.gov/IAGA/vmod/igrf.html

- a PYTHON code :

https://pypi.org/project/geomag/#files

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