

**data:**

$$\begin{aligned}
& S_k^{gl}, S_k^{gu} \quad \forall k \in G \\
& S_l^{cl}, S_l^{cu} \quad \forall l \in C \\
& c_{2k}, c_{1k}, c_{0k} \quad \forall k \in G \\
& a_l, b_l, c_l \quad \forall l \in C \\
& v_i^l, v_i^u \quad \forall i \in N \\
& v_i^l, v_i^u \quad \forall i \in N_{dc} \\
& S_i^d, Y_i^s \quad \forall i \in N \\
& P_i^d, Y_i^s \quad \forall i \in N_{dc} \\
& Y_{ij}, b_{ij}^c, T_{ij} \quad \forall (i, j) \in E \\
& Y_{ij} \quad \forall (i, j) \in E_{dc} \\
& s_{ij}^u, \theta_{ij}^{\Delta l}, \theta_{ij}^{\Delta u} \quad \forall (i, j) \in E \\
& p_{ij}^u \quad \forall (i, j) \in E_{dc} \\
& \mathbf{r} \\
& p_{dc}
\end{aligned}$$

**variables:**

$$\begin{aligned}
& S_k^g \quad \forall k \in G \\
& S_l^c \quad \forall l \in C \\
& P_l^{c,dc} \quad \forall l \in C \\
& V_i \quad \forall i \in N \\
& V_i \quad \forall i \in N_{dc} \\
& S_{ij} \quad \forall (i, j) \in E \cup E^R
\end{aligned}$$

(1)

$$\text{minimize: } \sum_{k \in G} c_{2k} (\Re(S_k^g))^2 + c_{1k} \Re(S_k^g) + c_{0k} \quad (2)$$

subject to:

$$\angle V_r = 0 \quad (3)$$

$$\mathbf{S}_k^{gl} \leq S_k^g \leq \mathbf{S}_k^{gu} \quad \forall k \in G \quad (4)$$

$$\mathbf{v}_i^l \leq |V_i| \leq \mathbf{v}_i^u \quad \forall i \in N \quad (5)$$

$$\sum_{k \in G_i} S_k^g + \sum_{l \in C_i} S_l^c - \mathbf{S}_i^d - \mathbf{Y}_i^s |V_i|^2 = \sum_{(i,j) \in E_i \cup E_i^R} S_{ij} \quad \forall i \in N \quad (6)$$

$$S_{ij} = \left( \mathbf{Y}_{ij}^* - \mathbf{i} \frac{\mathbf{b}_{ij}^c}{2} \right) \frac{|V_i|^2}{|\mathbf{T}_{ij}|^2} - \mathbf{Y}_{ij}^* \frac{V_i V_j^*}{\mathbf{T}_{ij}} \quad \forall (i, j) \in E \quad (7)$$

$$S_{ji} = \left( \mathbf{Y}_{ij}^* - \mathbf{i} \frac{\mathbf{b}_{ij}^c}{2} \right) |V_j|^2 - \mathbf{Y}_{ij}^* \frac{V_i^* V_j}{\mathbf{T}_{ij}^*} \quad \forall (i, j) \in E \quad (8)$$

$$|S_{ij}| \leq \mathbf{s}_{ij}^u \quad \forall (i, j) \in E \cup E^R \quad (9)$$

$$\theta_{ij}^{\Delta l} \leq \angle(V_i V_j^*) \leq \theta_{ij}^{\Delta u} \quad \forall (i, j) \in E \quad (10)$$

$$\mathbf{S}_l^{cl} \leq S_l^c \leq \mathbf{S}_l^{cu} \quad \forall l \in C \quad (11)$$

$$\sum_{k \in G_i} P_k^g + \sum_{l \in C_i} P_l^{c,dc} - \mathbf{P}_i^d - \mathbf{Y}_i^s |V_i|^2 = \sum_{(i,j) \in E_{i,dc} \cup E_{i,dc}^R} P_{ij} \quad \forall i \in N_{dc} \quad (12)$$

$$P_{ij} = p_{dc} \mathbf{Y}_{ij} \cdot (V_i^2 - V_i V_j) \quad \forall (i, j) \in E_{dc} \quad (13)$$

$$|P_{ij}| \leq \mathbf{p}_{ij}^u \quad \forall (i, j) \in E_{dc} \cup E_{dc}^R \quad (14)$$

$$P_l^c + P_l^{c,dc} = a + b |I_l^c| + c |I_l^c|^2 \quad \forall l \in C \quad (15)$$

$$|V_i|^2 |I_l^c|^2 = (S_l^c)^2 \quad \forall l \in C_i \quad \forall i \in N \quad (16)$$

$$(17)$$