

D[1 / ((x^2 - 1)^2 + x^2 / Q^2), x]

$$-\frac{\frac{2x}{Q^2} + 4x(-1 + x^2)}{\left(\frac{x^2}{Q^2} + (-1 + x^2)^2\right)^2}$$

Solve[% == 0, x]

$$\left\{\{x \rightarrow 0\}, \left\{x \rightarrow -\frac{\sqrt{-1 + 2Q^2}}{\sqrt{2}Q}\right\}, \left\{x \rightarrow \frac{\sqrt{-1 + 2Q^2}}{\sqrt{2}Q}\right\}\right\}$$

$$1 / ((x^2 - 1)^2 + x^2 / Q^2) /. x \rightarrow \frac{\sqrt{-1 + 2Q^2}}{\sqrt{2}Q}$$

$$\frac{1}{\frac{-1 + 2Q^2}{2Q^4} + \left(-1 + \frac{-1 + 2Q^2}{2Q^2}\right)^2}$$

Simplify[%]

$$\frac{4Q^4}{-1 + 4Q^2}$$

Series[(x^2 - 1)^2 + x^2 / Q^2, {x, Sqrt[1 - 1 / 2 / Q^2], 4}]

$$\left(-\frac{1}{4Q^4} + \frac{1}{Q^2}\right) + \left(4 - \frac{2}{Q^2}\right) \left(x - \sqrt{1 - \frac{1}{2Q^2}}\right)^2 + 4\sqrt{1 - \frac{1}{2Q^2}} \left(x - \sqrt{1 - \frac{1}{2Q^2}}\right)^3 + \left(x - \sqrt{1 - \frac{1}{2Q^2}}\right)^4 + O\left[x - \sqrt{1 - \frac{1}{2Q^2}}\right]^5$$